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"Transit Scheduling"

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TRANSIT SCHEDULING

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ABSTRACT

This paper provides an overview of the transit operational planning process with an emphasis on new methodologies in scheduling. The transit scheduling system usually comprises three interrelated components: (1) creation of timetables; (2) scheduling vehicles to trips; and (3) assignment of drivers. These three components are described in the paper with particular emphasis on the first component, because of its practical nature. The design of transit timetable is discussed from both a practical and an analytical viewpoint. In addition, a methodology is presented on construction of alternative computerized public timetables, based on procedures which improve the correspondence of vehicle departure times with passenger demand. The vehicle scheduling procedure is viewed through the minimization of the number of vehicles required to carry out a fixed or variable timetable. Finally, in describing the crew scheduling component, different approaches are briefly discussed. The overview and methodologies presented in the paper suggest that most of the scheduling tasks can be performed automatically or in a conversational man-computer mode. The adoption of new scheduling procedures will undoubtedly produce more efficient timetables as well as vehicle and crew schedules.

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TRANSIT SCHEDULING

Avishai Ceder

1. INTRODUCTION AND OVERVIEW

Any transit process planning includes four basic components performed in sequence: (1) network route design; (2) setting timetables; (3) scheduling vehicles to trips; and (4) assignment of drivers. It is desirable for all components to be planned simultaneously to exploit the system's capability to the greatest extent, and to maximize the system's productivity and efficiency. However, this planning process is extremely cumbersome and complex, and therefore seems to require separate treatment of each component, with the outcome of one fed as an input to the next component. The overview of this planning process is shown in Figure 1, with an emphasis on the scheduling components to be described in this paper. The second component in Figure 1 is aimed to meet the general public transportation demand. The demand varies during the hours of the day, the day of the week, from one season to another, and even from one year to another. This demand reflects the business, industrial, cultural, educational, social

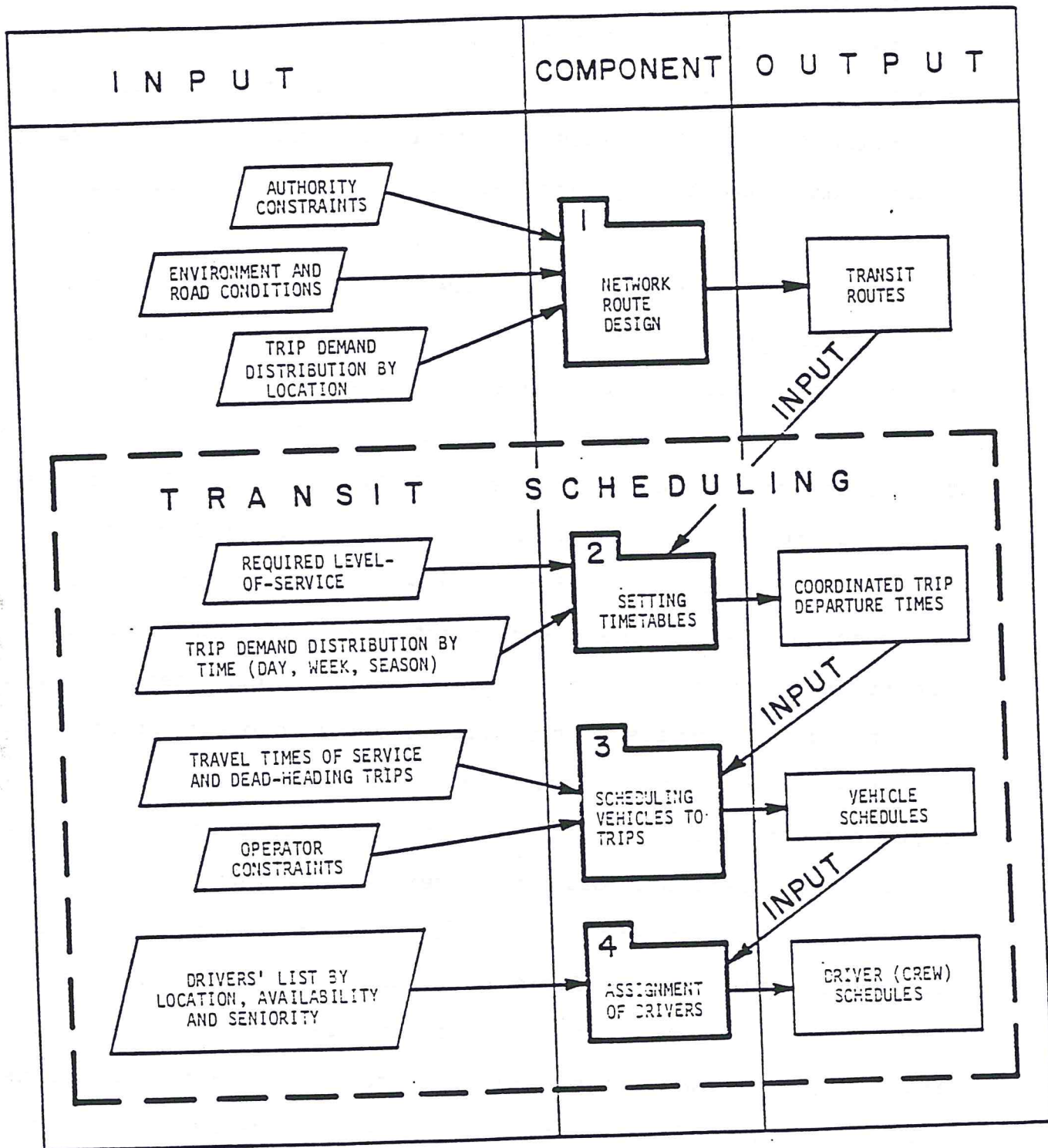


Figure 1: The Components of the Transit Operational Planning Process

and recreational transportation needs of the community. It is the purpose of this component to set appropriate timetables for each transit route to meet the variation in the public demand. Determination of timetables is performed on the basis of passenger counts, and must comply with service frequency constraints.

The third component in Figure 1 is to schedule vehicles to trips according to given timetables. A transit trip can be either planned to transport passengers along its route or to make a dead-heading trip in order to connect efficiently two service trips. The scheduler's task is to list all daily chains of trips (some dead-heading) for a vehicle, ensuring the fulfilment of the timetable requirements and the operator requirements (refueling, maintenance, etc.). The major objective of his task is to minimize the number of vehicles required. The fourth component in Figure 1 is to assign drivers to the outcome of vehicle scheduling. This assignment must comply with some constraints which usually are dependent of labour contract and refer to priority and rotation rules, rest periods, drivers' preferences, duty splitting and length etc. Any transit company, which naturally wishes to utilize its resources more efficiently, has to deal with problems encountered by various pay scales (regular, overtime, weekends, etc.), and by human-oriented dissatisfaction. Both components 3 and 4 in Figure 1 are very sensitive to internal and external factors -- sensitivity which could easily lead toward an inefficient solution.

The complexity involved in the transit operational planning process challenges researchers to develop automated computerized procedures. In the last ten years, a considerable amount of effort has been invested in this computerization task in order to provide more efficient controllable and responsive schedules. The best summary as well as the accumulated knowledge of this effort was presented in the second (Leeds, U.K., 1980), third (Montreal, Canada, 1983), and fourth (Hamburg, W. Germany, 1987) International Workshops on Vehicle and Crew Scheduling, and appear in the professional papers in the books by Wren (1981), Rousseau (1985) and Daduna and Wren (1988).

The overall view of the automated computerized procedures can be described by Figure 2, and in more detail by Figures 3 and 4. In the detailed description, the scheduling system architecture in Figure 3 is arranged by : input component, computation component, storage files, and the output component. In Figure 4, the summary and evaluation modules are described by : storage (internal) files, external input component, computation component, and the output component. The general description of the input data requirement by time of day and day of week is listed below and numbered according to the numbers appearing in Figs. 3 and 4.

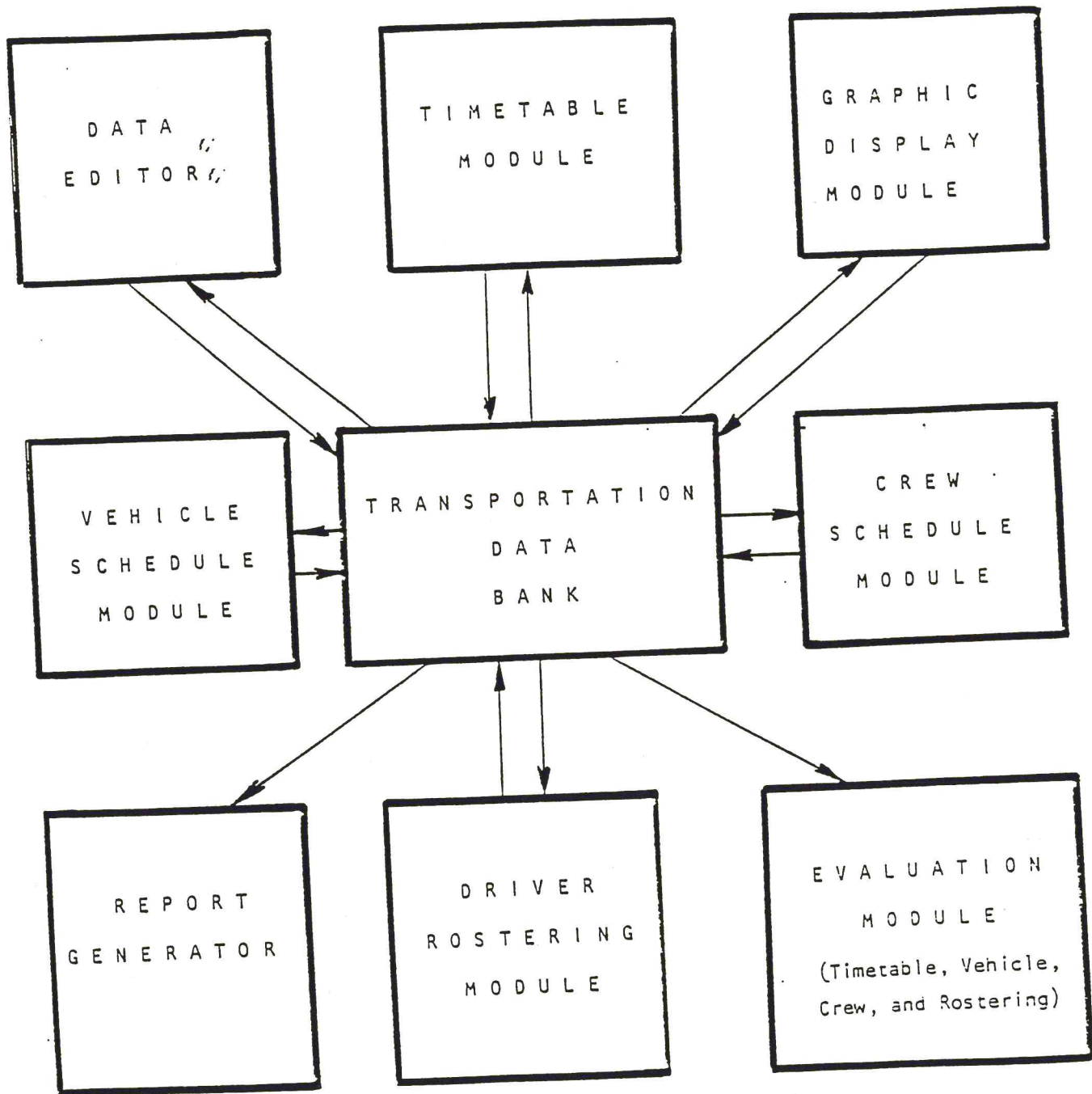


Figure 2: Overall View of the Transportation Scheduling System

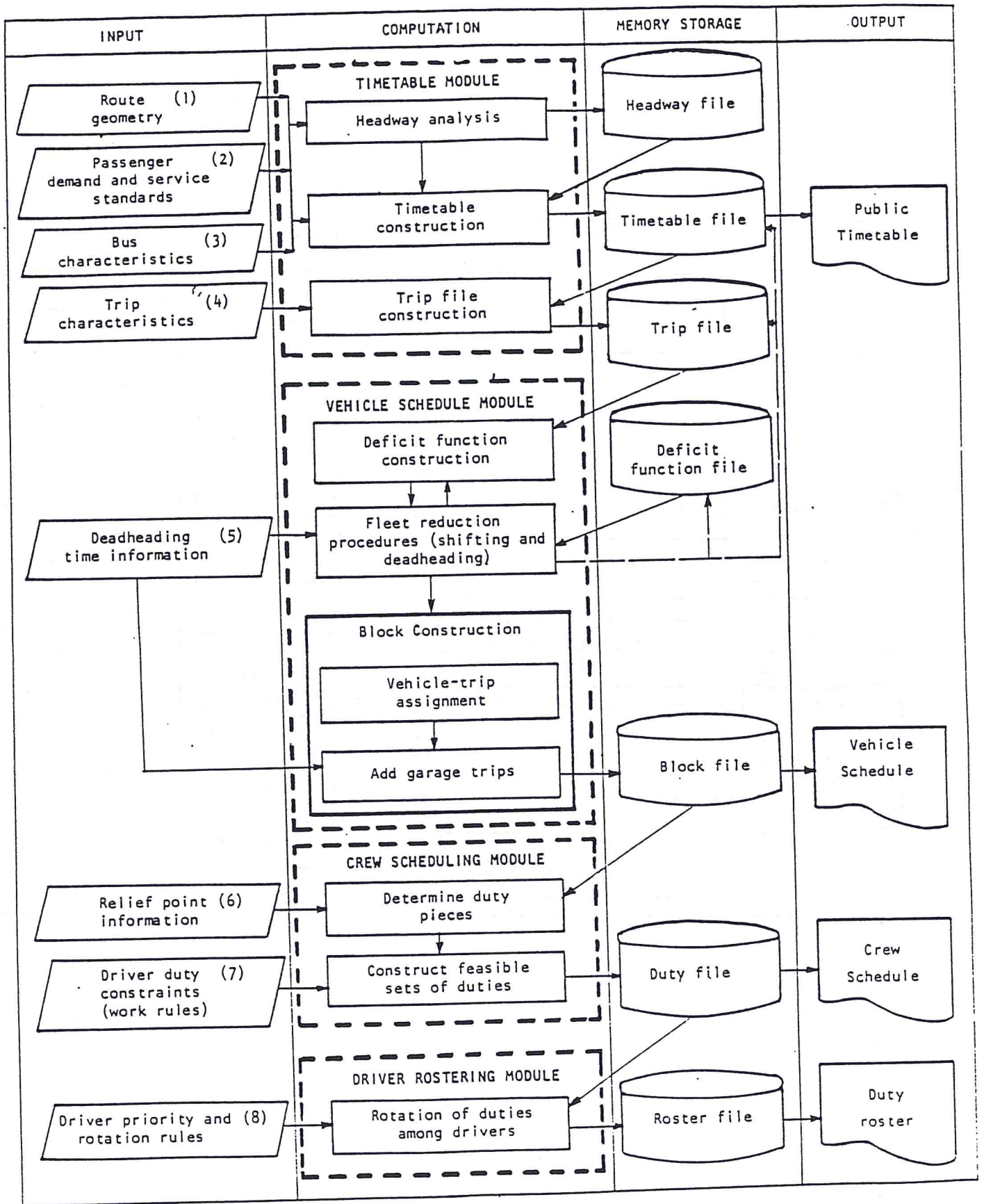


Figure 3: Functional Diagram of the Transportation Scheduling System (System Architecture)

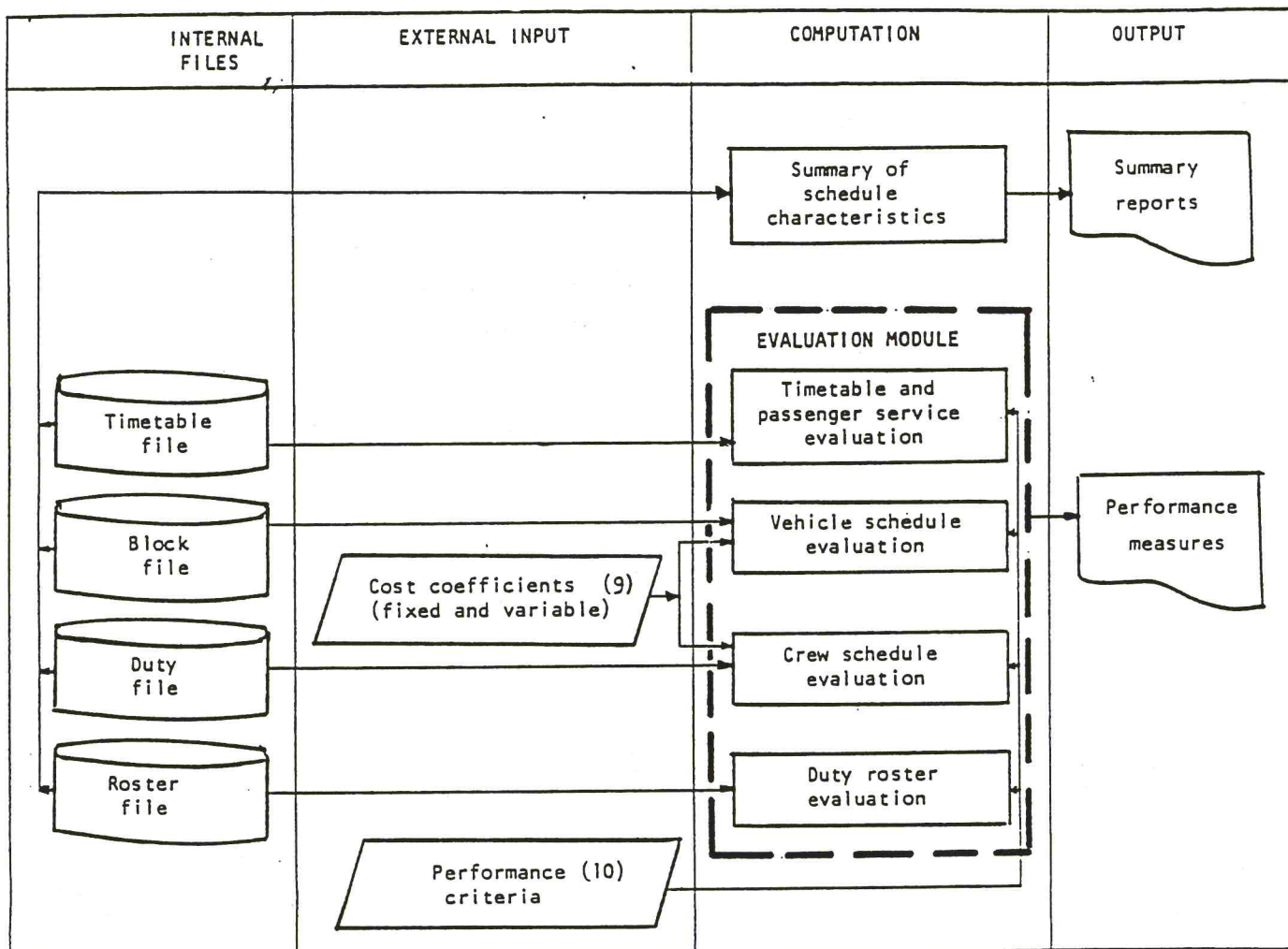


Figure 4: Performance Summary and Evaluation Modules

1. Route Geometry

- a) route number
- b) nodes -- stops and timepoints on a route
- c) pattern -- sequence of nodes on a route

2. Passenger Demand and Service Standards

- a) passenger loads between adjacent stops on a route
- b) load factor -- desired number of passengers on board the bus
- c) policy headway -- minimum frequency

3. Bus Characteristics

- a) bus type
- b) bus capacity
- c) running time -- bus travel time between stops and/or timepoints on a route

4. Trip Characteristics

- a) trip layover time (maximum and minimum)
- b) trip departure time tolerances (maximum departure delay and maximum departure advance)

5. Deadheading Time Information

- a) list of garages -- name and location
- b) list of trip start and end location
- c) deadheading times from garage locations to each trip start location (pull-outs)
- d) deadheading times from trip end locations to garage locations (pull-ins)
- e) deadheading time matrix between all trip end and start location

6. Relief Point Information

- a) relief point location (stops, trip start and end points and garages)
- b) interrelief point travel time

7. Driver Duty Constraints (dependent of labor contract)

- a) type of duty (early, late, split, full, tripper, etc.)
- b) duty length (maximum spread time)
- c) number of bus changes on duty
- d) meal breaks
- e) duty composition
- f) other work rules

8. Driver Priority and Rotation Rules

- a) list of drivers by name and type (e.g., part-time, full-time, years of each garage)
- b) driver priority or equality rules
- c) work day on and off pattern

9. Cost Coefficients
 - a) vehicle cost (fixed and variable)
 - b) driver cost (fixed and variable)
 - c) service benefit
 - d) other costs

10. Performance Criteria
 - a) measures of passenger service
 - b) performance measures for vehicle and crew schedules
 - c) performance measures for evaluating duty rosters
 - d) other criteria

2. FREQUENCY DETERMINATION AND TIMETABLE SETTING

2.1. Optimization Approaches

The best known simple theory on the problem of setting frequencies on bus routes is the square root rule described by Mohring (1972). Major weaknesses of the square root rule, which explain its total lack of acceptance by the industry, are that (i) it does not consider the effects of bus capacity constraints; (ii) it assumes ridership is fixed and independent of the service frequency provided; and (iii) it does not allow for minimum service level constraints.

Mathematical programming methods to determine bus frequency have been proposed by several researchers, for example, (Furth and Wilson, 1981; Koutsopoulos, Odoni and Wilson, 1985; Ceder and Stern 1984). The objective in Furth and Wilson is to maximize the net social benefit, consisting of ridership benefit and wait time savings, subject to constraints on total subsidy, fleet size, and acceptable levels of loading. Koutsopoulos et al. build up this formulation by explicitly incorporating crowding discomfort costs in the objective function and treating the time dependent character

of transit demand and performance. Their initial problem comprises a non-linear optimization program which can be solved efficiently only using linear approximations. Ceder and Stern address the problem with an integer programming formulation and a heuristic person-computer interactive procedure. The latter approach focuses on reconstructing timetables when the available vehicle fleet is restricted.

Several models have been developed for the simultaneous choice of routes and frequencies (Lampkin and Saalmans, 1967; Silman, Barzily and Passy, 1974; Last and Leak, 1976). The frequency determination components of these models typically minimize passenger wait time subject to capacity constraints under an assumption of fixed demand. A much more complex mathematical programming approach is proposed by Scheele (1977) to find optimal bus service frequencies in the long run case where the distribution of trips (but not total trip generation and production) is allowed to vary in response to service frequency provided. Finally, work by the Volvo bus planning group (Hasselstrom, 1981) has resulted in a package for choosing routes and frequencies.

Most of these models and theories are designed for a one-time application when the entire transit network is redesigned -- by definition a major and infrequent undertaking. They can also be useful in policy analysis, for example in the development of service policies and vehicle procurement. However, none have been accepted for repeated use by transit operators, partially because

of their orientation to only large-scale system change (they are not sensitive to a great variety of system specific operational constraints), and partially because they are either highly complex and hard to use, or over-simplified and hard to believe.

2.2. Practice in Timetable Preparation

Passenger demand at the route-level is generally gathered at one or more selected stops along the route where the bus carries its heaviest loads (point check). A more comprehensive method is based on load profile and running time information gathered along the entire length of the bus route (ride check). While the point checks are typically conducted several times a year, ride checks are often performed only once or twice during the year.

The methods commonly used by bus properties world-wide to construct timetables are based on the following criteria:

- (i) adequate space will be provided to meet passenger demand;
- (ii) an upper bound is placed on the headway to assure a minimum frequency of service.

The first requirement is generally covering for heavy ridership hours (peak periods), and the second for light ridership hours. These two requirements are often included in the service standards which have been widely adopted by many operators.

These service standards are a result both of codification of

existing traditional rules of thumb and a statement of policy. As such, they do not always accurately reflect decisions made by schedulers who are responsible for preparing the timetables. Based on a survey of transit service standards (Attanucci, Jaeger and Becker, 1979), the most frequently used procedures to derive bus timetables are:

policy headway -- maximum allowed service interval (lower bound on the frequency), used by virtually all operators, usually set to 15, 20, 30 or 60 minutes.

peak load factor -- the required number of buses is obtained by dividing the maximum observed passenger flow by a load factor (e.g., number of seats).

Examples of policy headway and load factor standards appear in Table 1.

Other methods used to determine especially the upper bound on the bus frequency are revenue/cost ratio (rough measure of efficiency and equity in the distribution of service) and vehicle productivity (measured in terms of passengers per vehicle-mile (or kilometer) or per vehicle-hour).

In current practice, schedule changes are performed using a mix of manual and computer generated reports with computerized reports being used in many large bus properties (e.g., SCRTD-Los Angeles, TTC-Toronto, EGGED-Israel). The procedure employed by SCRTD to develop timetables will be used here as an example. Based

Table 1: Examples of Service Standards for Timetable Preparation

Type of Standard	Bus Property	Period	Standard for Regular Routes
Policy headway	MBTA (Boston)	peak period	30 minutes
		mid-day	30 minutes
		evening	60 minutes
	Egged (Israel)	peak and off-peak periods	20 minutes
	TTC (Toronto)	mid-day	15 minutes
		evening	20 minutes
Load factor	MBTA	highest 30-min. of peak period	140% seating capacity
		peak period	120% seating capacity
		mid-day and evening	100% seating capacity
	Egged	peak period	130% seating capacity
		off-peak period	100% seating capacity
	TTC	peak and off-peak periods	100% seating capacity

on ride and point check data, the following steps are performed by the SCRTD scheduling department:

1. running times are established for each route by time of day (using the most recent ride check data);
2. the calculated bus speeds are examined for each time period and route segment (in order to correct special cases of speeding-up and slowing-down of buses, e.g., the drivers may speed-up toward the end of the route in order to extend their layover time);
3. headways are determined at the peak flow point (usually this is the time point at which the maximum passenger flow is observed; a time point is generally a bus stop at a major intersection or facility which appears on the public timetable);
4. departure (passage) times are set at the peak flow point;
5. corresponding departure times are set at all time points on the route including the terminals by using the established running times and the departure times at the peak point;
6. the departure (passage) times are adjusted to take into account two additional considerations: trips with short turns and the vehicle block construction procedure;
7. the final route timetable is completed;
8. following any revision of the schedule, the changes (or the new timetable) are marked on the timetable print instruction sheet which is transferred to the marketing department.

The scheduling files used by SCRTD consist of data from about 40,000 trips which is collected manually by checkers and then keypunched. All of the timetable and run cutting tasks are performed manually with work sheets prepared for key entry and computer processing. About 40% of the scheduler's time is devoted to data entry and proofreading computer generated reports.

It is interesting to note that the current procedures to create timetables are almost identical to those used about 40 years ago!. This observation is revealed in a classic reference on manual scheduling practices in 1947 (which was recently reprinted by the American Transit Association (Rainville, 1982). Nevertheless, today there are several advanced computer-based scheduling systems which were designed to overcome the cumbersome and complex problems in assigning vehicles and crew to given timetables. Some of these computerized systems also produce timetables, as discussed briefly below.

A well-known scheduling system in North America is RUCUS II (Luedtke, 1983) which is the latest interactive version of a driver RUn CUtting and Scheduling system, first released by the U.S. Department of Transportation in 1974. The bus timetable in RUCUS II is part of its required input and consists of the time a given trip passes the maximum load (flow) point. Multiple trips are created by using a single headway so that the resulting timetable is based on equal headways, although it is also possible to add or delete departures at the maximum load point or at other designated stops. In an earlier version of RUCUS, there is a component called TRIPS (Hinds, 1979) designed to assist in preparing headway sheets and timetables.

The most widely implemented computer-based scheduling system

in the United Kingdom is called BUSMAN (Williamson, 1983). Its operations' component is based on two programs developed at the University of Leeds called TASC and VAMPEIRS (Wren, 1971; Smith and Wren, 1981; Hartley and Wren, 1983). The first program TASC creates a timetable based on the headway file information. This file includes for each period of the day, the departure time of the first trip, the headway, and the time until which this headway applies.

The third computer scheduling program is HASTUS developed at the University of Montreal (Blais and Rousseau, 1983). The mathematical aspects of HASTUS are especially intended for crew scheduling. It is based on equal headways while smoothing the headways in the transition between time periods. Another interesting and partially computerized program is in operation in Amsterdam (Berkhout, 1983). This program constructs the timetable manually on clock-time charts (clock-times are arranged in a table form) while considering the different round trip times during the day. The computer simply generates the clock-time tables (each column is a clock time list in decreasing order with a 1-minute interval) so that the time difference between two adjacent columns is equal to the round trip time. The scheduler's duty is then to mark down the departure and layover times on the clock-time chart for each vehicle. In other words, this procedure attempts to overcome difficulties in timetable preparation by using a visual representation of possible trip times.

Generally speaking, the scheduling departments at various bus properties are seeking improvements at three different but interrelated levels:

1. elimination of time-consuming manual steps;
2. improved accuracy;
3. cost saving and productivity gains.

The first improvement is anticipated to take place in the relatively near future due to the introduction of computers into most scheduling departments. However, it is understood that even with the computerized process, many decisions will be made based on the scheduler's judgment (e.g., the development of timetables for periods with special activities, such as sporting events). The second improvement is directly related to the data collection methods. With greater use of automatic data collection systems, it is anticipated that this improvement could easily be attained. The third improvement is related to new and more efficient scheduling methods; the data collected should provide a reliable base for the scheduler's decision. For example, the automatic data collection systems may provide the required data, but without appropriate statistical models the data would be meaningless. The statistical models should accurately reflect the variations of both the passenger demand the vehicle performance measures.

2.3. Alternative Timetables

In this section, there is a review of the methods described by Ceder (1984, 1986). Figure 5 contains the description in a flow-chart fashion of the procedures to create bus frequencies and timetables. The first set of procedures consists of four methods described by Ceder (1984), that can be summarized in the following four equations:

(a) Two Point Check Methods for Time Period j

$$\text{Method 1: } (\text{Frequency})_j = \left[\frac{(\text{Load at the daily max. load point})_j}{(\text{Desired Occupancy})_j} \right] \quad (1)$$

$$\text{Method 2: } (\text{Frequency})_j = \left[\frac{(\text{Load at the hourly max. load point})_j}{(\text{Desired Occupancy})_j} \right] \quad (2)$$

(b) Two Ride Check Methods for Time Period j

$$\text{Method 3: } (\text{Frequency})_j = \text{MAX} \left[\frac{(\text{Area under the load profile in passenger - km.})_j}{(\text{Desired Occupancy})_j \times (\text{Route Length})_j}, \frac{(\text{Load at the hourly max. load point})_j}{(\text{Bus Capacity})_j} \right] \quad (3)$$

Method 4:

$(\text{Frequency})_j =$ same as Method 3 but

subject to a constraint that limits the length of the route

over which the load may exceed the product:

$$(\text{Frequency}) \times (\text{Desired Occupancy}) \quad (4)$$

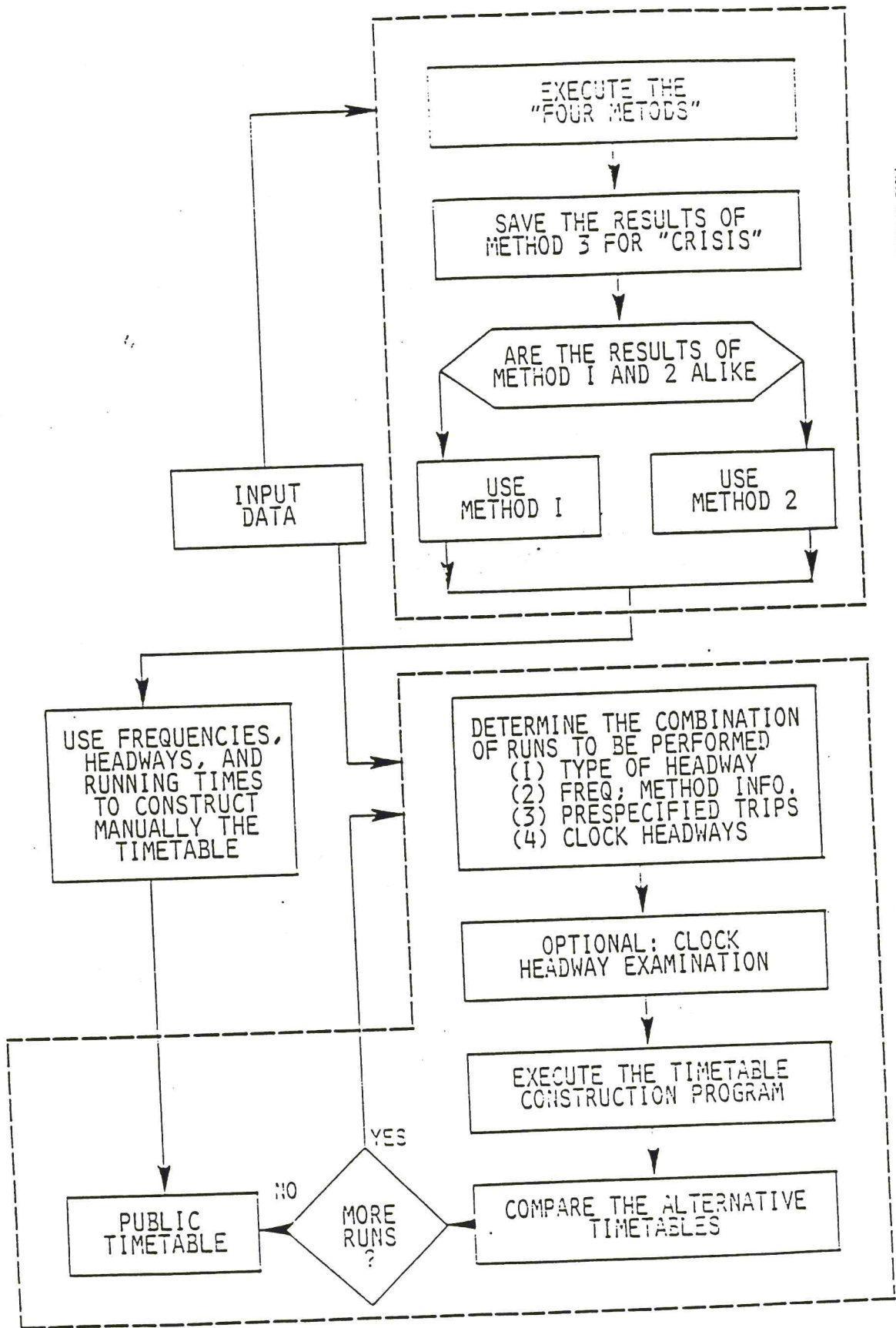


Figure 5: A Flow-Chart to Describe the Sets of Procedures for Generating Automated Transit Timetable

The first method is based on data gathered at one point during the whole day. This point is usually determined from old ride check data or from information given by a mobile supervisor. It represents the stop with the heaviest daily load along the route. The second method is based on the maximum load observed in each time period (usually an hour) rather than the whole day.

The third method is based on load profile information. The load profile is plotted with respect to the distance traveled from the departure point. Thus, the area under this curve serves as a productivity measure in passenger-kilometers. This area divided by the route length is the average load as opposed to the max. load in each period j in Method 2. Method 3 also guarantees, in an average sense, that the passengers on-board on the max. load segment will not experience crowding above the given bus capacity (number of seats + max. allowable standees). This method is useful for situations in which the scheduler wishes to know the number of bus runs he can save by raising the desired occupancy standard without incurring overcrowding. However, Method 3 can result in unpleasant travel for an extended distance in which the average load is above the desired occupancy. To control this undesirable situation, it is possible to establish a level of service criterion by restricting the total route distance having loads greater than the desired occupancy. This is in essence Method 4.

The second set of procedures in Figure 5, described by Ceder

(1986), analyses alternative ways for generating public timetables. Current timetable construction procedures provide the basis to establish the spectrum of alternative timetables. Three categories of options can be identified: (i) selection of type of headway; (ii) selection of a method or combination of methods for the setting of frequencies; and (iii) selection of special requests. These three groups of options are illustrated in Figure 6.

In the first category, alternative types of headways are considered. An equal headway simply means constant time intervals between adjacent departures in each time period, or the case of evenly spaced headway. A balanced headway refers to unevenly spaced headways in each time period, so that the observed passenger loads in all buses are similar. A smoothed headway is simply an average headway between the equal and the balanced headways. It is an option in cases where the available data are not sufficient for concrete conclusions about balanced headways, but at the same time, the scheduler believes that equal headways will result in significantly uneven loads. Such uneven load situations occur around work and school dismissal times and for trips with short turns.

In the second category, it is possible to select different frequency or headway determination methods. It allows for the selection of one method as well as combinations of methods for different time periods. The methods considered are the two point check and two ride check methods (Eqs. 1-4). In addition, there

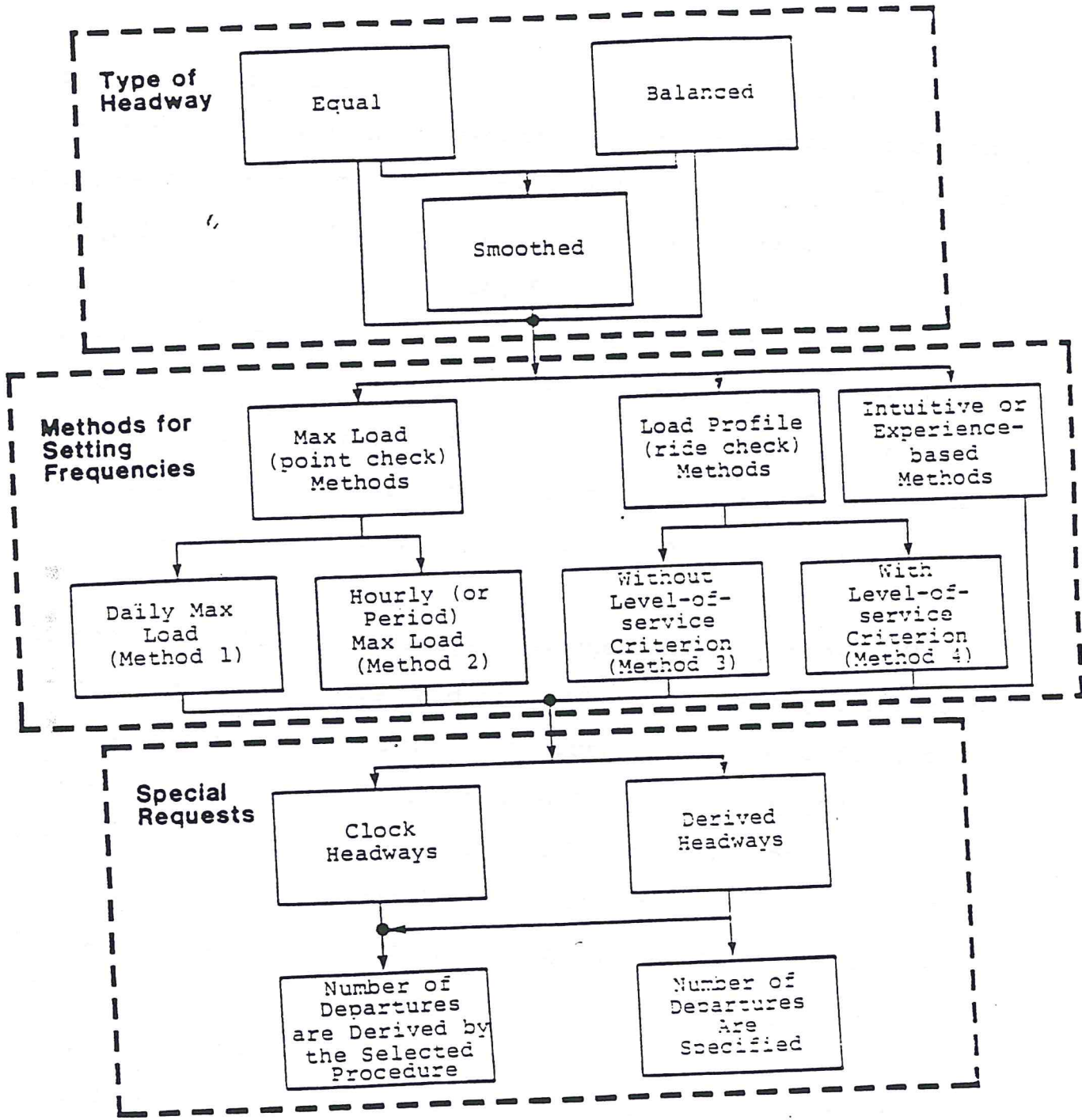


Figure 6: Alternative Timetables

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might be procedures used by the scheduler which are not based on data, but rather on observations made by the road supervisors and inspectors, as well as other sources of information.

The third category allows for special scheduling requests. One characteristic of existing transit timetables is the repetition of departure times, usually every hour. These easy-to-memorize departure times are based on the "clock headways" : (1,2,3,4,5),6,7,5,10,12,15,20,30,40,45 and 60 minutes. Note that headways less than 6 minutes are generally not considered by schedulers to influence the timing of passenger arrivals to a bus stop. However, for a general timetable construction procedure, there might be peak periods in which the headways are less than 6 minutes but need to be marked explicitly on the timetable.

The second possible special request is to allow the scheduler to prespecify the total number of bus departures during the time period. This request is most useful in crises where the scheduler needs to supply a working timetable for operation based on tightly limited resources (buses and/or drivers). By using his intuition, and controlling the total number of departures, the scheduler may achieve better results than by simply dropping departures without any systematic procedure. Also, there might be cases in which the scheduler would like to increase the level of service by allowing more departures. Such situations occur when there is a belief that passenger demand can be increased by providing improved (more frequent) service. Certainly, the latter special request can also

be approached through varying the desired occupancy values, and it is up to the scheduler to decide whether to control the passenger load or the number of departures which directly govern the required fleet size.

3. VEHICLE SCHEDULING

3.1. Objectives

This third component in Figure 1 determines, in an optimal manner, the construction of chains of trips or blocks (the vehicle schedule). The main objective of this function is to construct vehicle blocks while either:

- a) Using the existing number of vehicles (while minimizing total deadheading kilometers and disruption to the timetable) or
- b) Minimizing the number of vehicles required to carry out the schedule (the trip schedule).

In addition, the assignment of vehicle chains to garages should be determined in an efficient manner.

The attainment of these objectives can be carried out through the interaction of the following sub-functions:

- a) Trip characteristics study;
- b) Deadheading trip construction;
- c) Intertrip deadheading trip insertions;

- d) Timetable shifting;
- e) Vehicle trip chaining;
- f) Garage chain assignment.

In the literature edited by Wren (1981), Rousseau (1985), and Daduna and Wren (1988), one can find various approaches regarding how to automate the vehicle scheduling function. One of these approaches, based on an interactive graphical method, is described below.

3.2. Deficit Function Method

The deficit function method is a special graphical representation of the trip timetable for improving or generating new vehicle schedules. It is based on a theory developed by Ceder and Stern (1981) called Deficit Function Theory.

A deficit function is simply a step function which increases by one at the time of each trip departure and decreases by one at the time of each trip arrival. Such a function may be constructed for each terminal in a multi-terminal transit system. To construct a set of deficit functions, the only information needed is the transit timetable. The main advantage of the deficit function is its visual nature.

Let $d(k,t)$ denote the deficit for point k at time t . This point k can be either a terminal or a timepoint provided that some trips are initiated and/or terminated at this point. The

value of $d(k,t)$ represents the total number of departures less the total number of trip arrivals up to and including time t . The maximal value of $d(k,t)$ over the schedule horizon is designed $d(k)$. It is possible to partition the schedule horizon of $d(k,t)$ into a sequence of alternating hollow and maximal intervals. The maximal intervals define the interval of time over which $d(k,t)$ takes on its maximum value. A hollow interval is defined as the interval between two maximal intervals. Hollows may consist of only one point, and if this case is not on the schedule horizon boundaries, the graphical representation of $d(k,t)$ is emphasized by a clear dot.

If we denote the set of all the route and points (terminals or timepoints) as E , the sum of $D(k)$ for all k belongs to E is equal to the minimum number of vehicles required to service the set E . Mathematically, for a given fixed schedule:

$$N = D(k) = \max d(k,t) \quad (5)$$

where N is the minimum number of vehicles to service the set E .

When deadheading (DH) trips are allowed, the fleet size may be reduced below the level described in Eq. (5). Ceder and Stern (1981) describe this procedure based on the construction of a Unit Reduction Deadheading Chain (URDHC). Such a chain is comprised of a set of non-overlapping DH trips which, when inserted into the schedule, reduces the fleet size by one. The procedure continues inserting URDHC's until no more can be inserted or a

lower bound on the minimum fleet is reached. Determination of the lower bound is detailed in Stern and Ceder (1985). The deficit function theory for transit scheduling is extended by Ceder and Stern (1983), to include possible shifting in departure times within bounded tolerances.

A simple example on how the trip time can affect the fleet size at a given terminal is shown below. This example which is illustrated in Fig. 7, comprises 10 trips between three terminals m, u, v, and is shown along with a deficit function for terminal m. The trip numbers are circled and the departure and arrival terminals are indicated. According to the fleet size formula, 4 buses (minimum), are required at m. The 4 trip-chains are arranged in Fig. 7 in a similar fashion to that of the Gantt charts (follow the dashed lines).

Let us presume that the departure time is fixed, and that the trip time of trip number 7 can be reduced from 5 to 4 time-units. Then, the maximum value of the deficit function will be reduced by one, and only 3 buses will be required to satisfy the schedule according to the following 3 trip chains : [1-2-8], [3-4-5], [6-7-9-10]. On the other hand, if trip number 7 indeed falls into off-peak hours and its trip time can be reduced, one might find out that trip number 2 falls into peak hours and its trip time should be extended. In the latter case, the maximum value of the deficit function will again approach the value of 4 at time = 15 and no reduction in the fleet size could be achieved. Certainly, if the

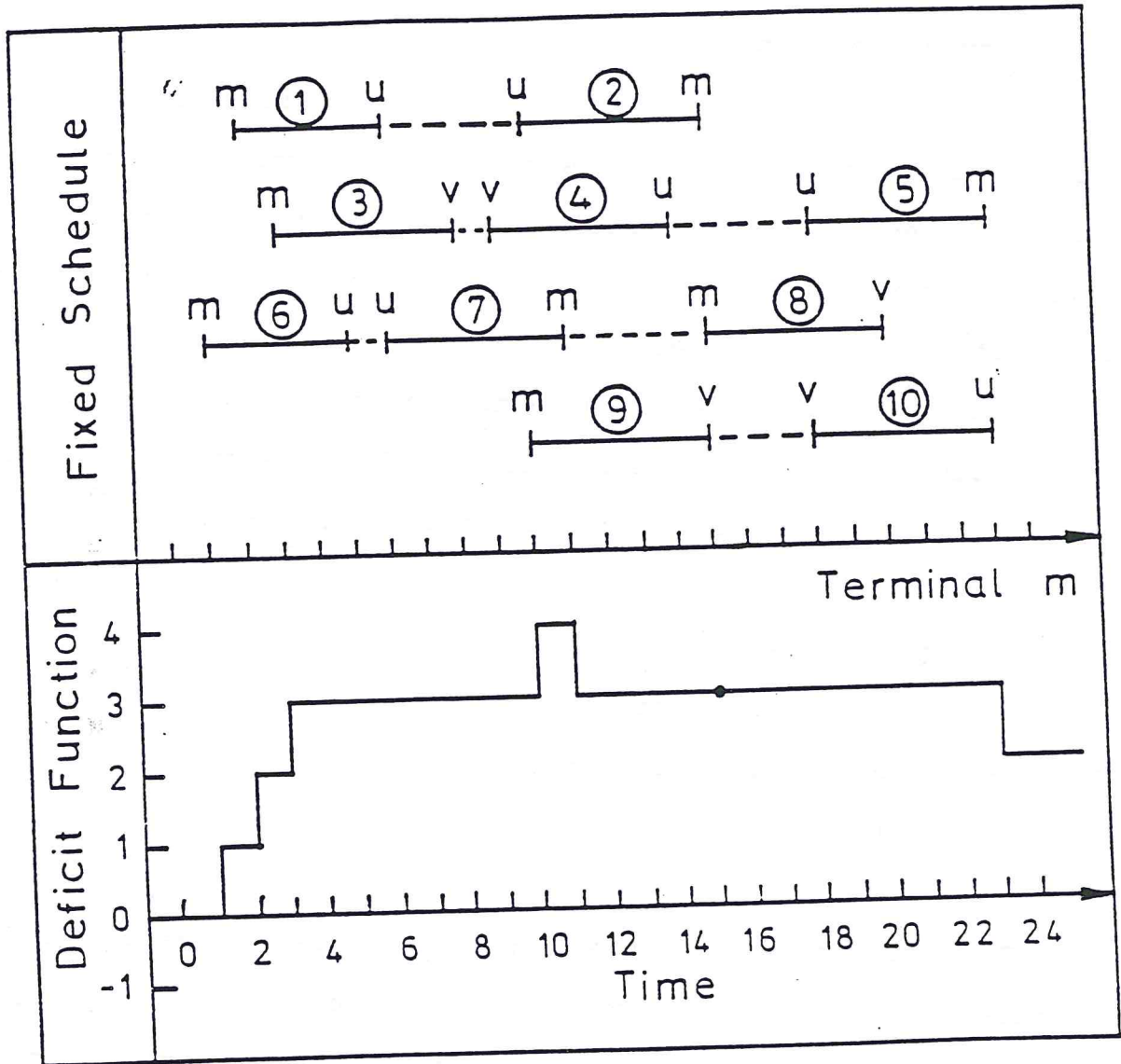


Figure 7: An Example of 10 Trips Arranged in Four Chains According to the Maximum Deficit Function Value

bus is running behind schedule, it can influence the remaining trips in its chain, depending on the recovery time available at the end of each trip. This example demonstrates how the deficit function can serve as a guide for examining possible changes in trip times of particular trips.

4. CREW SCHEDULING

The last component of the transit operational planning process (shown in Figure 1) is the assignment of drivers to carry on the vehicle schedules. The purpose of this assignment function is to determine a feasible set of driver duties in an optimal manner. The criteria for this determination is based on an efficient use of manpower resources while maintaining the integrity of any work rule agreements. The construction of the selected crew schedule is usually a result of the following sub-functions:

- a) duty piece analysis;
- b) work rules coordination;
- c) feasible duty construction;
- d) duty selection.

The duty piece analysis function divides or partitions each vehicle block at selected relief points into a set of duty pieces. These duty pieces are reassembled in the feasible duty construction function. Other required information is: interrelief point travel times and a list of relief points designated as required duty stops and start locations.

Theoretically, each relief point may be used to split the vehicle block into new duty-pieces. Usually, it is more efficient to use one or more of the following criteria for the selection of which relief points to include:

- a) Minimum duty piece length;
- b) Select a piece so that the next relief point selected is as close as possible to the maximum duty part time (maximum time before having a break);
- c) Only few, (say two) relief points in each piece;
- d) Operator decisions.

In order to utilize any crew scheduling method, a list of work rules to be used in the construction of feasible driver duties is required. The work rules are the result of an agreement between the drivers (or their unions) and the transit company (and/or public authorities).

The determination of different feasible sets of duties may be selected based on, for example, one or more of the following performance measures:

- a) Number of duties (drivers);
- b) Number of split duties;
- c) Total number of changes;
- d) Total duty hours;

- e) Average duty length;
- f) Total working hours;
- g) Average working time;
- h) Number of short duties;
- i) Costs.

The combinatorial nature of the crew scheduling problem requires various mathematical treatments, mostly associated with operations research methods. The literature appearing in Wren (1981), Rousseau (1985), and Daduna and Wren (1988) covers all these methods in detail.

In General, the mathematical treatment can be divided into heuristic and mathematical programming methods. The objective of the heuristic methods is to produce good solutions in a reasonable amount of computer time, while the objective of the mathematical programming methods is to produce optimal solutions. Both approaches can be executed either in automatic or interactive mode. In the last Computer-Aided Transit Scheduling Workshop detailed in Daduna and Wren (1988), one can find that most of the crew scheduling implementations are based on heuristic methods, and only a few are based on mathematical programming procedures (e.g., in Quebec City and Hamburg).

5. CONCLUDING REMARK

In practical transit scheduling, schedulers attempt to create timetables, allocate vehicles and crew in the most efficient manner possible. However, these scheduling tasks are time-consuming and exacting, requiring the services of imaginative and experienced schedulers. The overview and methodologies presented in this paper suggest that most of the scheduling tasks can be performed automatically or in a conversational man-computer mode.

The adoption of new scheduling methods will undoubtedly produce more efficient timetables, vehicle and crew schedules, which ultimately will result in saving operational cost. Moreover, it will then be possible to better match the transit demand which varies systematically by season, day-of-the-week, time-of-day, location and direction of travel, with the resultant transit services.

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